Exam. Code : 103203
Subject Code : 1113

B.A./B.Sc. 3rd Semester

MATHEMATICS

Paper—I (Analysis)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Do any **FIVE** questions selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Prove that a monotonically increasing sequence is convergent if and only if it is bounded above. Moreover it converges to l.u.b. of range.

   (b) Prove that sequence defined by :

   $x_1 = \sqrt{7}, x_{n+1}\sqrt{7 + x_n}$ is convergent and converges to positive root of :

   $y^2 - y - 7 = 0.$

2. (a) State and prove Cauchy's first theorem on limits.

   (b) Prove that the sequence $\{x_n\}$ where $x_n = \sum_{i=1}^{n} \frac{1}{i}$ is not convergent by showing that it is not Cauchy's sequence.

SECTION—B

3. (a) Test the convergence and divergence of the series :

$$\frac{2^2}{3^2}+\frac{2^2}{3^2}\frac{4^2}{5^2}+\frac{2^2}{3^2}\frac{4^2}{5^2}\frac{6^2}{7^2}+\ldots\ldots\ldots$$

(b) If the sequence $\{a_n\}$ is monotonic and converges to zero, then the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ is convergent.

4. (a) Prove that the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$ is convergent if and only if $p > 1$.

(b) Discuss the convergence of the series :

$$1+\frac{2x}{3^2}+\frac{3^2x^2}{3!}+\frac{4^3x^3}{4!}+\ldots\ldots\ldots\infty, x>0 .$$

SECTION—C

5. (a) Prove that every continuous function on [a, b] is Riemann integrable on [a, b].

(b) Define a function f on [0, k] where k is positive integer as follows :

$$f(x)=\begin{cases}0 \text{ when x is an integer}\\ 1 \text{ otherwise}\end{cases}$$

Then f is Riemann integrable on [0, k].

6. (a) Prove that if $f(x) = \sin x$, $0 \le x \le \pi/2$ then f is Riemann integrable on $[0, \pi/2]$.

(b) If m and M are g.l.b. and l.u.b. of function f(x) on [a, b] then show that :

$$m(b-a) \le \underline{\int_a^b} f(x)\,dx \le \overline{\int_a^b} f(x)\,dx \le M(b-a).$$

SECTION—D

7. (a) Show that :

$$\int_0^1 x^{-\frac{1}{3}}(1-x)^{-\frac{2}{3}}(1-2x)^{-1}dx = \frac{1\beta\left(\frac{2}{3}, \frac{1}{3}\right)}{9^{\frac{1}{3}}}.$$

(b) Test the convergence of $\int_0^{\pi} \frac{\sqrt{x}}{\sin x}\,dx$.

8. (a) Using Gamma function evaluate :

$$\int_0^a x^4\sqrt{a^2 - x^2 dx}.$$

(b) Show that :

$$\int_0^{\pi} x^m (\csc x)^n \; dx \text{ exist if and only if } n < m + 1.$$